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MATHEMATICAL MODELING OF MOISTURE LOSS DURING APPLE CUBES DEHYDRATION

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Abstract
An important number of internal and external parameters influence drying behavior. Among the external parameters one can include temperature, velocity of the drying agent and relative humidity of the drying medium, while internal parameters include density, permeability, porosity, sorption–desorption characteristics and thermo physical properties of the fruits being dried.
Mathematical models have proved to be very useful in the design and analysis of processes occurring during fruits dehydration. All parameters used by simulation models are directly related to the drying conditions.
This study was aimed to establish a mathematical relation between the moisture content of apple fruits cubes of different thicknesses and two of the most important technological drying parameters: temperature and time drying period. Among the different investigated equations the results indicated that the Exponential Association model can present better predictions for describing the evolution of the obtained experimental data. The fit quality of the drying data to the proposed models was evaluated by means of statistical tests such as linear determination coefficient (R^2), relative absolute error (ε) and the reduced chi-square (χ^2). A good agreement with the experimental data was found. The comparisons and correlations of the results indicated that validation and performance of the established mathematical model is rather reasonable.

Keywords: apple, moisture content, dehydration, mathematical modeling

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1. INTRODUCTION

Dehydration is a very frequently used method of fruits preservation since it is responsible for the reduction of the product moisture content which greatly retards microbial and chemical deterioration (Vega-Galvez, A. *et al.*, 2008; Doymaz, I., Pala, M., 2003).
In order to improve the existing drying systems and the control of the process, it is important to dispose of accurate models to simulate the drying curves under different conditions (Toğrul İ.T., Pehlivan D., 2004).
Depending on the applied equations, Simal *et al.*, 2005 classify models as theoretical, semi-empirical and empirical. They mention also that from a strict sense, there are four prevailing transport phenomena involved in drying (internal and external heat transfer and internal and external mass transfer) which may describe the drying process. Nevertheless, the corresponding four classical partial differential equations demand considerable computing time for numerical solution. Therefore, various empirical models have been used to simulate fruit-drying processes. Among these, the most noteworthy are Henderson-Pabis (Doymaz I., Ismail O., 2011; Mancilla Y.N. *et al.*, 2011), Newton (Evin D., 2011; Thakur A.K. *et al.*, 2010), Lewis (Kaya A. *et al.*, 2007), Logarithmic (Uribe E. *et al.*, 2011; Lahsasni S. *et al.*, 2004), Page (Contreras C. *et al.*, 2008), Modified Page (Vega-Galvez, A. *et al.*, 2008; Menges H.O., Erteikin, C., 2006), Midilli-Kucuk (Thakur A.K. *et al.*, 2010).
The high temperatures and long time periods required for water removal from apple fruits may seriously affect their nutritional and organoleptic quality. For this reason, these technological parameters are considered as being critical variables (Vega A. *et al.*, 2007; Akpinar E.K., 2006; Doymaz I., 2005).
Modeling the drying behavior of apples requires statistical methods of linear and non-linear regression and correlation analysis.

In the present work, the constants and coefficients of the best fitting model involving the above mentioned drying variables were determined.

2. MATERIALS AND METHODS

Homogeneous lots of *Golden Delicious* variety apples, acquired in a local market in the city of Bacau, were selected according to their maturity, size and color and then washed in cold water and peeled. The apples without seeds were cut into cubes of 5, 10 and 15 mm in thickness. Samples of 10 g were submitted to hot air convective drying in a laboratory oven at 4 different temperatures: 60, 80, 100 and 120°C. After 5, 10, 20, 40, 60 and 120 minutes at each of the above mentioned drying temperatures the samples were weighed using an analytical balance with an accuracy of ± 0.0001 g in order to determine the quantity of water removed. The results were reported at 100 g of raw material.

Based on the experimental data and using Microsoft Excel™ 2007 spreadsheets and CurveExpert® software various empirical models have been used in order to establish the best equations able to correlate the moisture content of apples cubes with temperature and drying time. The fit quality of the experimental data to the proposed models was evaluated by means of statistical tests including linear determination coefficient (R^2), absolute relative error (ε) and the reduced chi-square (χ^2). The above parameters were calculated using the fallowing relations:

$$\varepsilon = \left| \frac{M_{exp} - M_{pred}}{M_{exp}} \right| \cdot 100 \quad [\%] \qquad (1)$$

$$\chi^2 = \frac{\sum_{i=1}^{N} (M_{exp} - M_{pred})^2}{N - z} \qquad (2)$$

where: M_{exp} – experimental moisture loss
M_{pred} – predicted moisture loss
N – number of experimental data
z – number of constants
i – number of terms
M_{exp} – experimental moisture loss
M_{pred} – predicted moisture loss

3. RESULTS AND DISCUSSION

3.1 Mathematical Model

The moisture ratio data apple cubes dried at different temperatures for different periods of time was fitted into various mathematical models. An exponential association was found to be the most appropriate mathematical model:

$$Y = A \cdot (1 - E(-B \cdot \tau)) \qquad (3)$$

where: Y – moisture content,
A, B – equation coefficients
τ – time drying period (minutes)

The coefficients of correlation are listed in Table 1.

Table 1. Coefficients values for equation 3

Apple cubes thickness [mm]	*t* [°C]	Coefficient *A*	Coefficient *B*	*R²*
5	60	91.894403	0.0332994	0.997989
	80	88.040436	0.0591384	0.998789
	100	78.432547	0.1759680	0.998836
	120	86.072376	0.2187030	0.998812
10	60	90.038230	0.0246428	0.996783
	80	88.016481	0.0341171	0.999385
	100	78.021985	0.1052767	0.999757
	120	85.830814	0.1687613	0.999860
15	60	117.19567	0.0100173	0.994971
	80	88.559115	0.0204248	0.998162
	100	76.699508	0.0654822	0.998399
	120	85.248750	0.0979098	0.997247

The coefficients A and B are related with the drying temperature (t in °C) by a 2nd degree polynomial equation (equation 4).

$$Coefficient = a \cdot t^2 + b \cdot t + c \qquad (4)$$

The coefficients values are related in Table 2. Combination of relations (3) and (4) and replacement of coefficients with numerical values conducted to the mathematical models final form (equations 3, 4 and 5) (Table 3).

Table 2. Coefficients values for equation 4

Apple cubes thickness [mm]	Coefficient	*a*	*b*	*c*	R^2
5	*A*	0,007184	-1,428422	152,8888	0,724814
	B	0,000011	0,001464	-0,10084	0,943419
10	*A*	0,006144	-1,219024	142,3497	0,599544
	B	0,000034	-0,003559	0,113166	0,982373
15	*A*	0,023241	-4,721904	317,0235	0,999288
	B	0,000014	-0,000934	0,014124	0,977641

Table 3. Mathematical models final form

Apple cubes thickness [mm]	Mathematical models	ε	χ^2
5	$Y = (0.007184 \cdot t^2 - 1.428422 \cdot t + 152.888758) \cdot (1 - EXP(-(0.000011 \cdot t^2 + 0.001464 \cdot t - 0.100835) \cdot \tau))$	0.833687	0.588834
10	$Y = (0.006144 \cdot t^2 - 1.219024 \cdot t + 142.349658) \cdot (1 - EXP(-(0.000034 \cdot t^2 + 0.003559 \cdot t - 0.113166) \cdot \tau))$	0.706907	0.311644
15	$Y = (0.023241 \cdot t^2 - 4.721904 \cdot t + 317.023462) \cdot (1 - EXP(-(0.000014 \cdot t^2 + 0.000934 \cdot t - 0.014124) \cdot \tau))$	0.356170	0.801110

The calculated values graphic representations are illustrated in Figures 1 – 3.

3.2 Statistical analysis of models

The evaluation of the fit quality of the experimental data to the employed models was achieved by using three important statistics which are widely used in mathematical simulations of food processing (R^2, ε and χ^2). High values for R^2 (> 0.9) were obtained for the three models at the four drying temperatures used. The absolute error between the calculated values and the experimental data was below 1%. The small values of reduced chi-square confirm that these equations satisfy the drying data adequately.

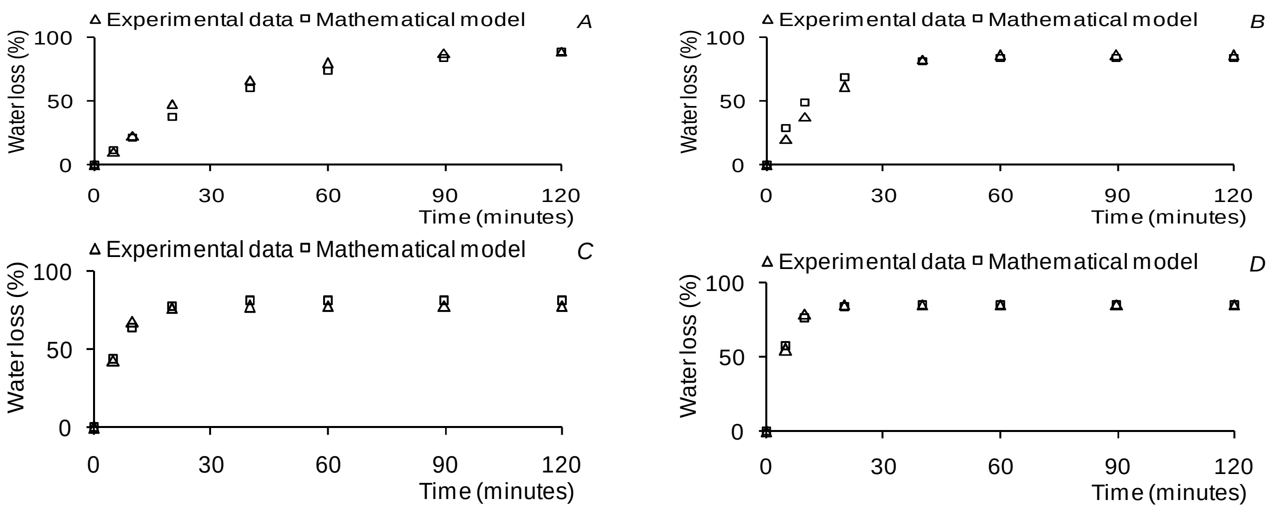

Figure 1. Moisture loss from 5 mm apple cubes during dehydration process at different temperatures (A – 60°C, B – 80°C, C – 100°C, D – 120°C)

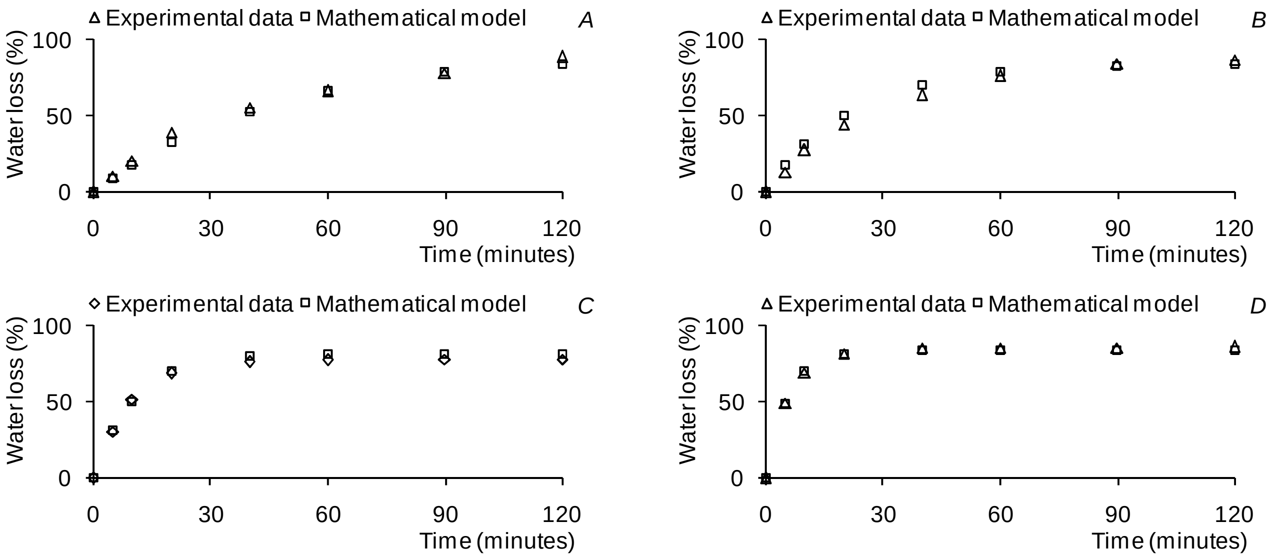

Figure 2. Moisture loss from 10 mm apple cubes during dehydration process at different temperatures (A – 60ºC, B – 80ºC, C – 100ºC, D – 120ºC)

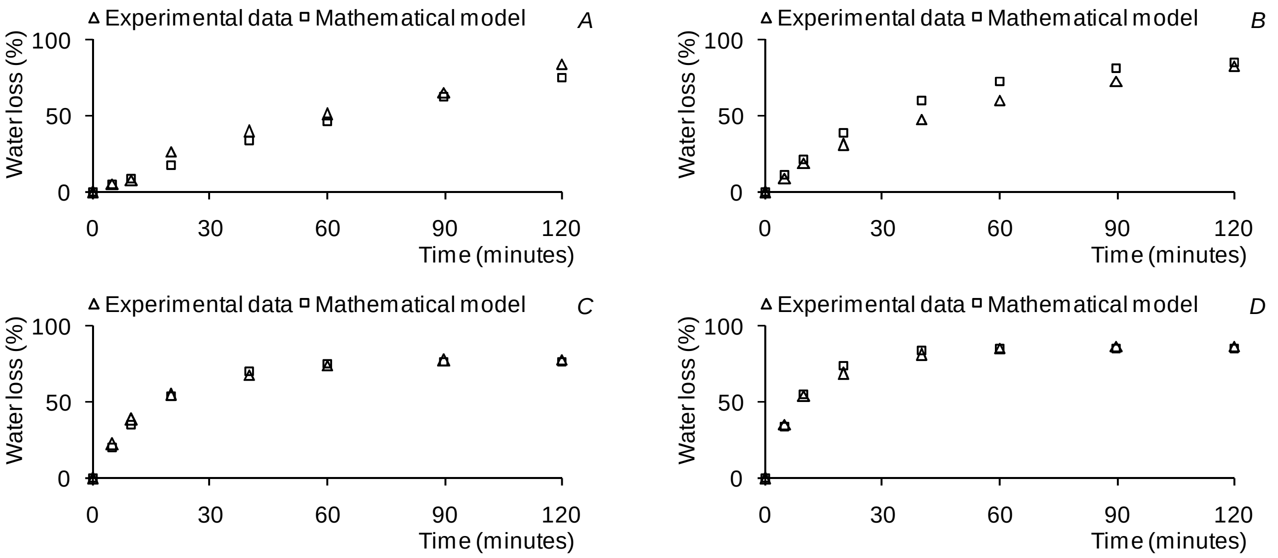

Figure 3. Moisture loss from 15 mm apple cubes during dehydration process at different temperatures (A – 60ºC, B – 80ºC, C – 100ºC, D – 120ºC)

4. CONCLUSIONS

In this investigation a mathematical model for apples drying has been developed.
Although the model developed is mathematically simple, it is able to provide reliable predictions concerning the influence of temperature and time drying period on the dried apple cubes moisture content evolution

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