COMPUTER AIDED GENERATION OF UNFOLDED CURVES OF PIPE JOINTS

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Abstract
In this paper is comparative studied the feasibility of the pipelines unfolding and construction of some surfaces, which are used in ventilation and air conditioning installations. To solve this application, in paper, we resort to methods of the descriptive geometry and the mathematical methods, using appropriate software for these applications.
The calculation of the unfolding has a wide applicability, especially in the connection of the pipes with equal or different diameters. This calculation can be done by using several methods among the descriptive geometry method or the analytical method. In this paper, using these methods, we had been analyzed four cylinders, calculating their unfoldings of the intersection curves. The first, we use the descriptive geometry method. For illustration, we took four cylinders with the following diameters: C_1, C_2 and C_3 of diameters $D_1 = D_2 = D_3 = 200$ mm and C_4 of diameter $D_4 = 400$ mm. We know the distance d=100 mm, between the axes of the cylinders C_3 and C_4, the angles $V=22.5^0$ and $U=135^0$. After, the curves were calculated using the mathematical program, in order to automatic trace their unfoldings. For this aim, the projection of the intersection curves necessitates the solving of the following phases: the writing of the curves equations resulted from the intersections of the areas that can be unfolded; the writing of the transformations equations by the unfolding of the intersection curve.
So, these equations were introduced in the mathematical program, to check the method of descriptive geometry.

Key words: unfolding, method, mathematical program, curve of intersection, cylinder.

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1. INTRODUCTION

In this paper is comparative studied the feasibility of the pipelines unfolding and construction of some surfaces, which are used in ventilation and air conditioning installations. To solve this application, in paper, we resort to methods of the descriptive geometry and the mathematical methods, using appropriate software for these applications.

Although this problem has been treated often in articles and studies, this paper extends the applications within such an area less studied, ventilation and air conditioning equipment and not least the approach of the problem.

2. MATERIALS AND METHODS

This calculation can be done by using several methods among the descriptive geometry method or the analytical method. The first, we use the descriptive geometry method. After, the obtained equations were introduced in the mathematical program, to check the method of descriptive geometry.

2.1 Descriptive geometry method

A key role, in achieving the efficient ventilation installations, has the sections and the configurations of the fluid transport pipelines. The drawing of the cylindrical surfaces unfoldings have difficulties in cutting as economic the material, template which should contain all dimensions necessary in the effective realization. In the cylindrical surfaces unfolding, in theory, the following problems are:

– unfolding configuration of the curve of the surface base;

– points characteristic of the section (inflexion, extreme, etc...);

– points of the base surface, or how many generators are chosen so that the unfolding is accurate;

– achievement of templates for material cutting as economic.

For illustration, we took four cylinders with the following diameters: C_1, C_2 and C_3 of diameters $D_1 = D_2 = D_3 = 200$ mm and C_4 of diameter $D_4 = 400$ mm (Fig. 1). We know the distance d=100 mm, between the axes of the cylinders C_3 and C_4, the angles $V=22.5^0$ and $U=135^0$. A 3 D view is illustrated in the Figure 2.

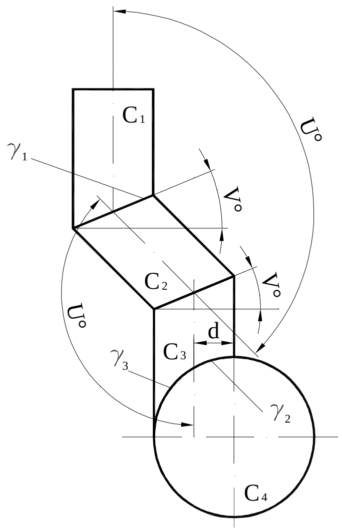

Figure 1: Geometrical elements of the cylinders

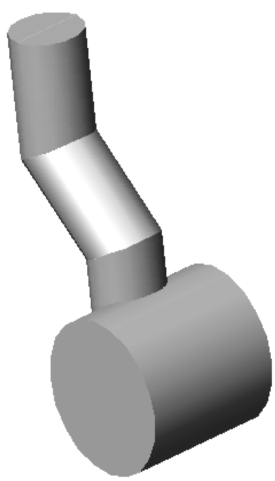

Figure 2: 3D view of the cylinder

2.1.1. Unfolding of the C_1 right circular cylinder (rotation)

Be rotating cylinder, of known dimensions (diameter d and height h, h_1) from Figure 3, with the base in the [H] horizontal plane, sectioned with a [P] end plane[2,3,4,5,7].

To obtain a fairly accurate unfolding, the base circle is divided into a greater number of points. The elements necessary to unfold are the generators lengths (which are vertical straight lines, having the actual dimension in the vertical projection) and the distances between generators, that are directly on the horizontal projection of the base circle. The cylinder being right circular, the normal plane on the generators is the plane base, so the base circle is the normal section.

The normal section on the generator has as transform a straight line, of length equal with the unfolded length of the cylinder circle of D_1 diameter, and the generators transforms will be perpendicular to the section transform in the corresponding points. On the straight line equal with the unfolded length $A_oB_o = \widehat{ab}$, $A_oC_o = \widehat{bc}$, etc. are measured. In the $A_o, B_o, ...A_o$ points, some perpendiculars will be constructed, which the $A_oA_{1o} = a'a'_1$, $B_oB_{1o} = b'b'_1$, etc. lengths of each generator, are measured The $A_{1o}, B_{1o}, ...A_{1o}$ curve is the transform of the section curve, made with the [P] plane in cylinder.

2.1.2. Unfolding of the C_2 oblique cylinder

Be the oblique cylinder, having the base a circle located in the [H] horizontal plane, of known dimensions (diameter, length generators, angle of generators to the base) (Fig. 1).

To determine the oblique cylinder unfolding (Fig. 5) must know the actual dimensions of the respective generators and the distances between them. The generators being front straight lines, the actual dimensions are in vertical projection.

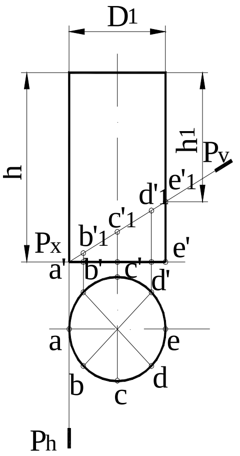

Figure 3: Right circular cilynder C_1

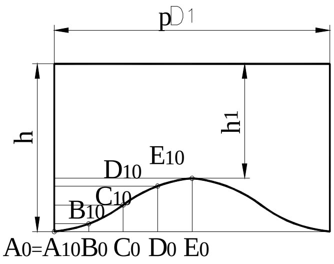

Figure 4: Unfolding of the C_1 right circular cilynder

The distance between generators is made by a normal section with the [P] plane, having the $P_h \perp O_1O_2$ horizontal mark and the $P_v \perp O'_1O'_2$ vertical mark. The normal section is the $1,2,...,8$ ellipse. The actual dimension of the ellipse, which the distance between the generators will be measured, are deducted by rabating the [P] plane on the [H] horizontal plane. The rabate axis is the [P] orizontal mark.

The $1_{1o},2_{1o},...,5_{1o}$ curve is the actual length of the ellipse, which by unfolding is a straight line (Fig.6). On a straight line $1_o2_o = 1_{1o}2_{1o,}\, 2_o3_o = 2_{1o}3_{1o}$, etc. are measured. From the $1_o,2_o,...,5_o$ points, the perpendiculars will be constructed, where $1_oA_o = 1'a', 1_oA_{1o} = 1'a'_1, 2_oB_o = 2'b'$, $2_oB_{1o} = 2'b'_1$, etc. will be measured. The $A_oB_oC_oA_o$ and $A_{1o}B_{1o}C_{1o}A_{1o}$ curves are the transforms of the two bases of the cylinder.

On these curves appear the C_o, C_{1o} inflexion points, and their symmetries, which are on the apparent contour generators from the [H] plane, contained by the tangent planes to the cylinder and perpendicular on the base plane.

From the unfolding of this cylinder, the portions that are not interested are removed, remaining the $M_oN_oP_oR_oS_o$ şi $M_{1o}N_{1o}P_{1o}R_{1o}S_{10}$ curves as unfolding, which are actually what we wanted to know.

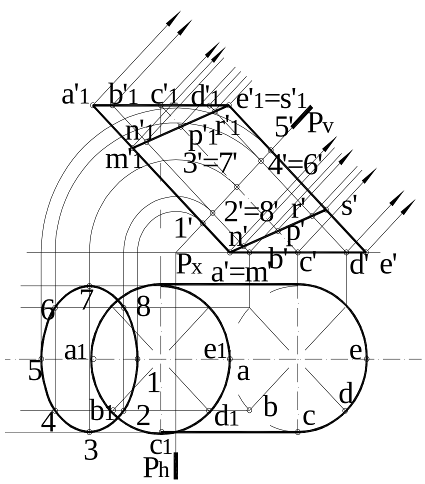

Figure 5: Oblique circular cylinder C_2

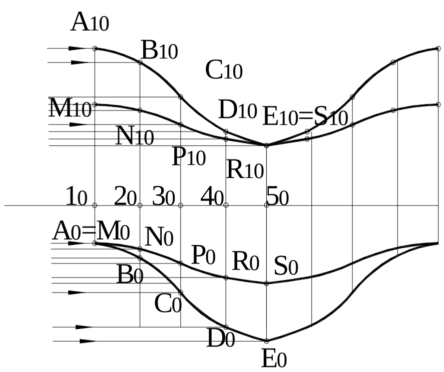

Figure 6: Unfolding of the C_2 right circular cilynder

2.1.3. Unfolding of the C_3 and C_4 cylinders

To determine the unfolding of the C3 and C4 cylinders,
auxiliary planes are constructed, similar to the above cases. The cylinders base is divided

into equal parts (Fig. 7), and through these divisions auxiliary planes are constructed, determining the points of the intersection curve (Fig. 8, 9). Obviously, the unfolding lengths are equal with the lengths of the base circle of D_3, D_4 diameters.

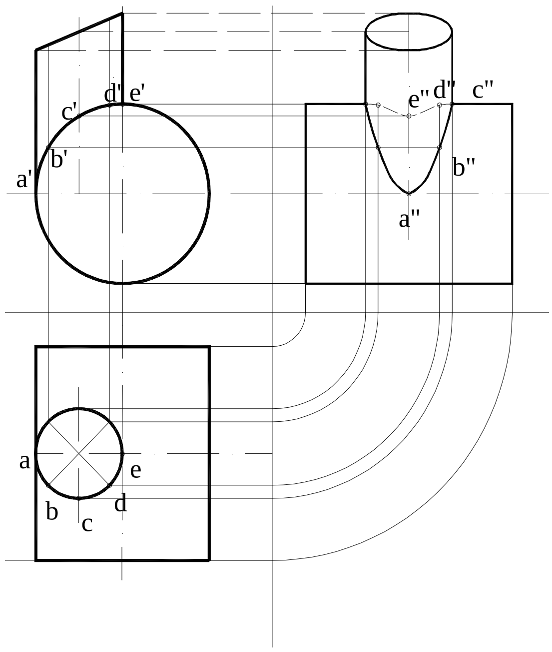

Figure 7: Circular C_3 si C_4 cylinder

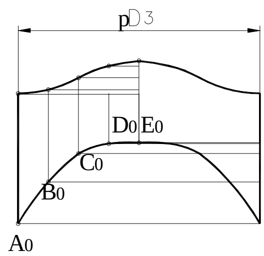

Figure 8: Unfolding of the C_3 cilynder

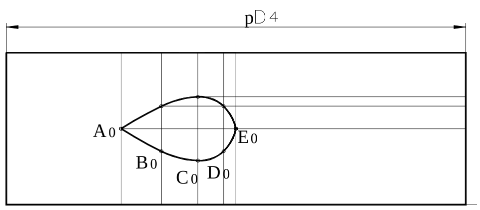

Figure 9: Unfolding of the C_4 cilynder

Obviously, the unfolding lengths are equal with the lengths of the base circle of D_3, D_4 diameters.

2.2. Analitical method

The projection of the intersection curves necessitates the solving of the following phases:

- the writing of the curves equations resulted from the intersections of the areas that can be unfolded;
- the writing of the transformations equations by the unfolding of the intersection curve.

2.2.1. The calculation of the intersection curve γ_1 of the cylinder C_1

In the Figure 10 the geometrical elements for the first cylinder are presented. Be a point M (m, m ') of coordinates x, y in plane and x_d,y_d in unfolding. The transformed equation of the vertical projection of the curve plotted on the cylinder in the equation of the unfolding curve can be carried out by expressing the coordinates x and y by x_d and y_d. Thus [1,6,8,9,10]:

$$y = R_1 \cdot \sin W \qquad (1)$$

$$x = x_d \qquad W \in [0, 2 \cdot \pi] \qquad (2)$$

$$\widehat{mt} = y_d = R_1 \cdot W \rightarrow W = \frac{y_d}{R_1}, \qquad (3)$$

so:

$$y = R_1 \cdot \sin \frac{y_d}{R_1} \qquad (4)$$

$$x = x_d \qquad (5)$$

The equation of the vertical projection of the obtained curve from the section of the right circular cylinder with an inclined plane at 22.5^0, is the equation of a straight line of a known gradient:

$$x = tg\, V \cdot (y + R_1), y \in [-R_1, R_1] \qquad (6)$$

For this cylinder, the equation of the transformation curve is obtained by applying the transformation (1), (2) to the equation (6). We obtain thus:

$$x_d = tg\, V \cdot (R_1 \cdot \sin \frac{y_d}{R_1} + R_1), y_d \in [0, 2 \cdot \pi \cdot R_1] \qquad (7)$$

For an angle V = 22.5^o and a cylinder radius R_1 = 100mm, we obtain the Figure 10, by introducing the relation (7) into mathematical program.

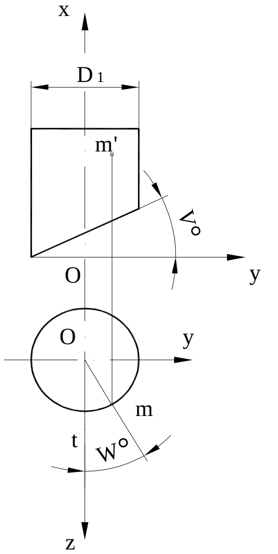

Figure 10: The geometrical elements of the cylinder C_1

The others curves of intersection, in this particular case, the angles being the same, have the same forms as in Figure 11.

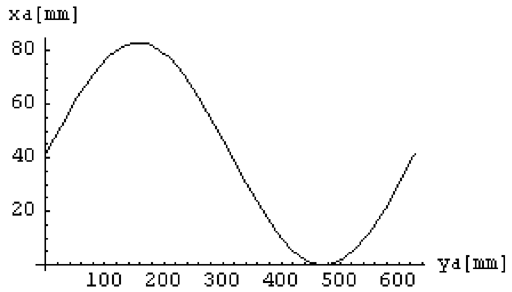

Figure 11: Curve unfolding γ_1 of the cylinder C_1

2.2.2. The calculation of the intersection curve γ_2 of the cylinder C_4

We refer to the straight circular cylinders C_3 and C_4 (Fig. 12). The chosen reference system is Oxyz. Be Q (q, q ', q') a point of x, z coordinates in plane and x_d,z_d in unfolding. The transformed equation of the vertical projection of the plotted curve on the cylinder, in the equation of the unfolded curve can be carried out by expressing the coordinates x and z by x_d and z_d.

$$x = R_4 \cdot \cos\alpha = R_4 \cdot \cos\frac{x_d}{R} \qquad (8)$$

$$z = z_d \qquad (9)$$

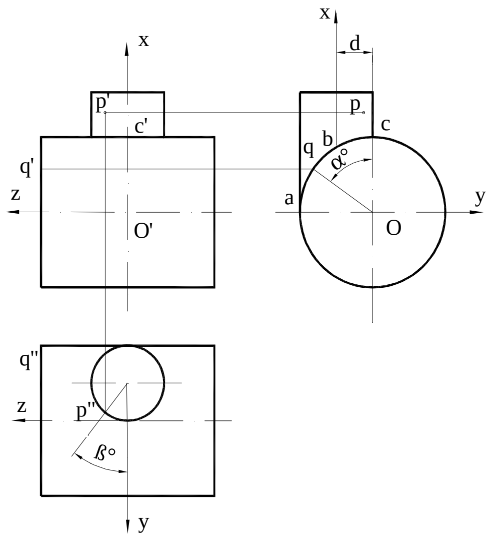

Figure 12: Elements of cylinders C_3 andC_4

The equations of the cylinders expressed in the chosen reference systems are:

$$C_1 : x^2 + y^2 = R^2, \qquad (10)$$

$$C_2 : z^2 + (y-d)^2 = r^2. \qquad (11)$$

Eliminating the variable y, we obtain the equation of the vertical projection of the intersection:

$$z^2 - x^2 + R_4^2 - R_3^2 + d^2 - 2 \cdot d \cdot \sqrt{R_4^2 - x^2} = 0 \qquad (12)$$

The equation of the intersection curve of the cylinder is obtained by appling (8, 9) relations to the (12) equation:

$$z_d^2 + \left(R_4 \cdot \cos\frac{x_d}{R_4}\right)^2 - R_3^2 = 0 \qquad (13)$$

$$z_d = \pm\sqrt{R_3^2 - (R_4 \cdot \sin\frac{x_d}{R_4} - d)^2} \qquad (14)$$

where:

$$x_d \in \left[-\widehat{bc}, \widehat{bo}\right], \qquad (15)$$

so:

$$x_d \in \{-R_4 \cdot arctg[(d-R_3)/R_4], R_4 \cdot arcsin[(d+R_3)/R_4]\} \qquad (16)$$

Replacing with the cylinders data we obtain the following curve (Fig.13).

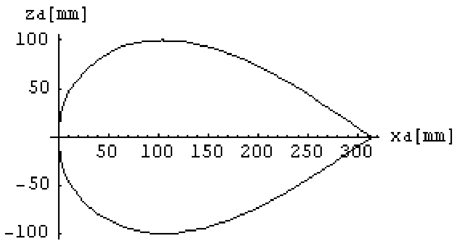

Figure 13: Curve unfolding γ_2 of the cylinder C_4

2.2.3. The calculation of the intersection curve γ_3 of the cylinder C_3

Taking the P (p, p ', p") point of x and z coordinates in plane and x_d and z_d in unfolding, we have:

$$x = x_d \,, \quad (17)$$

$$z = R_3 \cdot \sin\beta = R_3 \cdot \sin\frac{z_d}{R_3} \quad (18)$$

The equation of the unfolding curve of the C_3 cylinder will be obtained by applying the (17, 18) relations to the (12) equation:

$$x_d = \pm\sqrt{R_4^2 - (R_3 \cdot \cos\frac{z_d}{R_3} + d)^2}\,, \quad (19)$$

where:

$$z_d \in \left[-\pi \cdot R_3, \pi \cdot R_3\right] \quad (20)$$

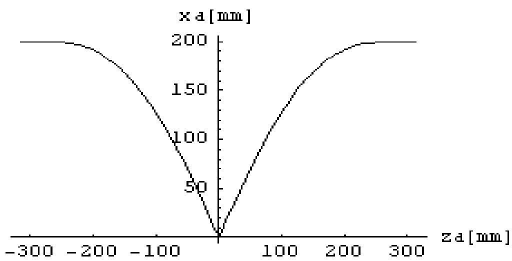

Figure 14: Curve unfolding γ_3 of the cylinder C_3

Replacing with the data cylinders, we obtain the curve from Figure 14.

3. CONCLUSIONS

A key role, in achieving the efficient ventilation installations, has the sections and the configurations of the fluid transport pipelines. The drawing of the cylindrical surfaces unfoldings have difficulties in cutting as economic the material, template which should contain all dimensions necessary in the effective realization. In all the unfolding curves we observe that they are the same, using the two methods.

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