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# MODELING OF THE THERMO-PHYSICAL PROPERTIES OF AQUEOUS SUCROSE SOLUTIONS I. DENSITY AND DYNAMIC AND KINEMATIC VISCOSITY

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***Abstract***
*Sucrose has been always used as a standard for calibration of densimeters, viscosimeters, refractometers, polarimeters etc. Density and viscosity of aqueous sucrose solutions are important both from the basic and practical point of views. They are used for laboratory studies (structure, nucleation and crystal growth), industrial practice (beverage, candying) and molecular biology investigations.*
*The aim of this study was to establish mathematical relations between temperature and sucrose concentration with density or dynamic and kinematic viscosity of aqueous sucrose solutions. In order to assess and select a suitable mathematical model the known data were fitted in different equations. Three equations were generated for density and one for viscosity, taking in consideration the level of precision and simplicity of formulation.*
*For density of aqueous sucrose solutions one equation with an average of relative errors of 0.00011% and two equations with an regression coefficients greater than 0.999 were generated, for ranges of 5 – 85% in sucrose concentration and temperature of 10 – 80 °C. For dynamic viscosity an equation was formulated with average relative errors of 0.11% for intervals of temperature of 0 to 100 °C and sucrose concentration of 16 to 80%. The kinematic viscosity was calculated as the ratio of the dynamic viscosity to the density.*

Keywords: sucrose, aqueous solutions, thermo-physical properties, density, viscosity

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## 1. INTRODUCTION

Density and viscosity of aqueous sucrose solutions are important both from the basic and practical point of views (Bubník, et al., 1995).
Although these thermo-physical properties of sucrose have been widely studied, there is always a need to gather the existing information dispersed in different sources and to introduce it in easy to use mathematical equations even though due to the molecular association in pure sucrose solutions the mathematical treatment of density and viscosity of aqueous sucrose solutions is rather complex (Vasserman et al., 2001).
Poling *et al.* (Poling et al., 2001) consider that an ideal system for the estimation of a physical property should: (1) provide reliable physical and thermodynamic properties for pure substances and for mixtures at any temperature, pressure, and composition, (2) require a minimum of input data, (3) choose the least-error route (i.e., the best estimation method), (4) indicate the probable error and (5) minimize computation time. Few of the available methods approach this ideal, but some serve remarkably well.
Therefore, the aim of the present study is to correlate the density and the viscosity of the aqueous sucrose solutions with their concentration and temperature in order to establish reliable mathematical models easy to use with simple computer software.

## 2. EXPERIMENTAL PROGRAM

Tabular data (Table 1 and 2) concerning the variation of aqueous sucrose solutions density and dynamic viscosity with sucrose concentration and temperature were used as primary data for the regression analysis.

Microsoft Excel™ 2007 spreadsheets, CurveExpert® and TableCurve 3D® v.4 software were used to establish the equations.

**Table 1. Density $\rho$ [kg/m$^3$] of aqueous sucrose solutions as a function of the mass fraction *X* [%] and temperatures *t* [°C] [4]**

| Sucrose concentration, *X* w [%] | Temperature, *t* [°C] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| 5 | 1019.56 | 1017.79 | 1015.03 | 1011.44 | 1007.14 | 1002.20 | 996.7 | 990.65 |
| 10 | 1040.15 | 1038.10 | 1035.13 | 1031.38 | 1026.97 | 1021.93 | 1016.34 | 1010.23 |
| 15 | 1061.48 | 1059.15 | 1055.97 | 1052.06 | 1047.51 | 1042.39 | 1036.72 | 1030.56 |
| 20 | 1083.58 | 1080.97 | 1077.58 | 1073.50 | 1068.83 | 1063.60 | 1057.85 | 1061.63 |
| 25 | 1106.47 | 1103.59 | 1099.98 | 1095.74 | 1090.94 | 1085.61 | 1079.78 | 1073.50 |
| 30 | 1130.19 | 1127.03 | 1123.20 | 1118.80 | 1113.86 | 1108.44 | 1102.54 | 1096.21 |
| 35 | 1154.76 | 1151.33 | 1147.28 | 1142.71 | 1137.65 | 1132.13 | 1126.16 | 1119.79 |
| 40 | 1180.22 | 1176.51 | 1172.25 | 1167.52 | 1162.33 | 1158.71 | 1150.88 | 1144.27 |
| 45 | 1206.58 | 1202.01 | 1198.16 | 1193.25 | 1187.94 | 1182.23 | 1176.14 | 1169.70 |
| 50 | 1233.87 | 1220.64 | 1224.98 | 1219.93 | 1214.50 | 1208.70 | 1202.56 | 1196.11 |
| 55 | 1262.11 | 1257.64 | 1252.79 | 1247.59 | 1242.05 | 1236.18 | 1220.99 | 1223.53 |
| 60 | 1291.31 | 1286.61 | 1281.59 | 1276.25 | 1270.61 | 1264.67 | 1258.45 | 1251.99 |
| 65 | 1321.46 | 1316.56 | 1311.38 | 1305.93 | 1300.21 | 1294.21 | 1287.96 | 1281.52 |
| 70 | 1352.55 | 1347.49 | 1342.16 | 1336.63 | 1330.84 | 1324.80 | 1318.55 | 1312.13 |
| 75 | 1384.58 | 1379.36 | 1373.98 | 1366.36 | 1362.52 | 1356.46 | 1350.21 | 1343.83 |
| 80 | 1417.50 | 1412.20 | 1406.70 | 1401.10 | 1395.20 | 1389.20 | 1383.00 | 1376.60 |
| 85 | 1451.30 | 1445.90 | 1440.50 | 1434.80 | 1429.00 | 1422.90 | 1416.80 | 1410.60 |

**Table 2. Viscosity $\mu$ [mPa] of aqueous sucrose solutions as a function of the mass fraction *X* [%] and temperatures *t* [°C] [[5, 6]]**

| Sucrose concentration, *X* w [%] | Dynamic viscosity, $\mu$ [mPa] at different temperature, *t* [°C] | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 95 | 100 |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| 16 | - | - | - | 1.24 | 1.02 | 0.84 | 0.70 | 0.62 | 0.52 | 0.50 | 0.48 | 0.46 |
| 18 | - | - | - | 1.37 | 1.11 | 0.91 | 0.77 | 0.65 | 0.55 | 0.52 | 0.49 | 0.46 |
| 20 | 3.78 | 2.64 | 1.95 | 1.49 | 1.18 | 0.97 | 0.81 | 0.68 | 0.59 | 0.53 | 0.50 | 0.47 |
| 21 | 3.98 | 2.77 | 2.03 | 1.56 | 1.23 | 1.00 | 0.84 | 0.71 | 0.61 | - | - | - |
| 22 | 4.19 | 2.90 | 2.12 | 1.62 | 1.28 | 1.04 | 0.87 | 0.73 | 0.63 | - | - | - |
| 23 | 4.42 | 3.05 | 2.22 | 1.69 | 1.33 | 1.09 | 0.90 | 0.76 | 0.65 | - | - | - |
| 24 | 4.66 | 3.21 | 2.33 | 1.77 | 1.39 | 1.13 | 0.93 | 0.79 | 0.67 | - | - | - |
| 25 | 4.93 | 3.38 | 2.45 | 1.85 | 1.45 | 1.17 | 0.97 | 0.82 | 0.70 | - | - | - |
| 26 | 5.22 | 3.57 | 2.57 | 1.94 | 1.52 | 1.22 | 1.01 | 0.85 | 0.72 | - | - | - |
| 27 | 5.54 | 3.77 | 2.71 | 2.04 | 1.59 | 1.28 | 1.05 | 0.88 | 0.75 | - | - | - |
| 28 | 5.89 | 3.99 | 2.86 | 2.14 | 1.66 | 1.34 | 1.10 | 0.92 | 0.78 | - | - | - |
| 29 | 6.27 | 4.23 | 3.02 | 2.25 | 1.74 | 1.40 | 1.14 | 0.96 | 0.81 | - | - | - |
| 30 | 6.69 | 4.49 | 3.19 | 2.37 | 1.83 | 1.47 | 1.20 | 1.00 | 0.85 | 0.71 | 0.67 | 0.62 |
| 31 | 7.15 | 4.77 | 3.38 | 2.50 | 1.93 | 1.54 | 1.25 | 1.04 | 0.88 | - | - | - |
| 32 | 7.65 | 5.08 | 3.58 | 2.65 | 2.03 | 1.61 | 1.31 | 1.09 | 0.92 | - | - | - |
| 33 | 8.21 | 5.43 | 3.81 | 2.80 | 2.14 | 1.69 | 1.37 | 1.14 | 0.96 | - | - | - |
| 34 | 8.84 | 5.81 | 4.05 | 2.97 | 2.26 | 1.78 | 1.44 | 1.19 | 1.00 | - | - | - |
| 35 | 9.54 | 6.23 | 4.32 | 3.15 | 2.39 | 1.87 | 1.51 | 1.25 | 1.05 | - | - | - |
| 36 | 10.31 | 6.69 | 4.62 | 3.35 | 2.53 | 1.98 | 1.59 | 1.31 | 1.10 | - | - | - |
| 37 | 11.19 | 7.21 | 4.95 | 3.57 | 2.69 | 2.09 | 1.67 | 1.37 | 1.15 | - | - | - |
| 38 | 12.17 | 7.79 | 5.32 | 3.82 | 2.86 | 2.21 | 1.76 | 1.44 | 1.20 | - | - | - |
| 39 | 13.27 | 8.44 | 5.72 | 4.08 | 3.04 | 2.35 | 1.86 | 1.52 | 1.26 | - | - | - |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 14.55 | 9.17 | 6.17 | 4.38 | 3.24 | 2.49 | 1.97 | 1.60 | 1.32 | 1.10 | 1.04 | 0.96 |
| 41 | 16.00 | 9.99 | 6.67 | 4.70 | 3.46 | 2.65 | 2.08 | 1.68 | 1.39 | - | - | - |
| 42 | 17.67 | 10.93 | 7.23 | 5.06 | 3.71 | 2.82 | 2.21 | 1.77 | 1.46 | - | - | - |
| 43 | 19.58 | 11.98 | 7.87 | 5.47 | 3.98 | 3.01 | 2.35 | 1.88 | 1.54 | - | - | - |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| 44 | 21.76 | 13.18 | 8.58 | 5.92 | 4.28 | 3.22 | 2.50 | 1.99 | 1.63 | - | - | - |
| 45 | 24.29 | 14.55 | 9.38 | 6.42 | 4.61 | 3.46 | 2.66 | 2.11 | 1.71 | - | - | - |
| 46 | 27.22 | 16.11 | 10.30 | 6.99 | 4.98 | 3.71 | 2.85 | 2.25 | 1.82 | - | - | - |
| 47 | 30.60 | 17.91 | 11.33 | 7.63 | 5.40 | 4.00 | 3.05 | 2.40 | 1.93 | - | - | - |
| 48 | 34.56 | 19.98 | 12.51 | 8.35 | 5.87 | 4.32 | 3.28 | 2.56 | 2.05 | - | - | - |
| 49 | 39.22 | 22.39 | 13.87 | 9.17 | 6.40 | 4.68 | 3.53 | 2.74 | 2.19 | - | - | - |
| 50 | 44.74 | 25.21 | 15.43 | 10.11 | 6.99 | 5.07 | 3.81 | 2.94 | 2.34 | 1.92 | 1.76 | 1.56 |
| 51 | 51.29 | 28.48 | 17.24 | 11.18 | 7.67 | 5.52 | 4.12 | 3.17 | 2.50 | - | - | - |
| 52 | 59.11 | 32.34 | 18.34 | 12.41 | 8.44 | 6.03 | 4.47 | 3.42 | 2.69 | - | - | - |
| 53 | 68.51 | 36.91 | 21.79 | 13.84 | 9.32 | 6.61 | 4.87 | 3.70 | 2.89 | - | - | - |
| 54 | 79.92 | 42.38 | 24.68 | 15.49 | 10.34 | 7.27 | 5.30 | 4.01 | 3.12 | - | - | - |
| 55 | 93.86 | 48.90 | 28.07 | 17.42 | 11.50 | 8.02 | 5.81 | 4.36 | 3.37 | - | - | - |
| 56 | 110.00 | 56.79 | 32.12 | 19.68 | 12.86 | 8.88 | 6.38 | 4.76 | 3.66 | - | - | - |
| 57 | 132.30 | 66.39 | 36.95 | 22.35 | 14.44 | 9.88 | 7.04 | 5.20 | 3.98 | - | - | - |
| 58 | 159.00 | 78.15 | 42.78 | 25.51 | 16.29 | 11.10 | 7.80 | 5.72 | 4.34 | - | - | - |
| 59 | 192.50 | 92.70 | 49.84 | 29.28 | 18.46 | 12.40 | 8.65 | 6.30 | 4.75 | - | - | - |
| 60 | 235.70 | 110.90 | 58.49 | 33.82 | 21.04 | 14.00 | 9.66 | 6.98 | 5.20 | 4.10 | 3.72 | 3.18 |
| 61 | 291.60 | 133.80 | 69.16 | 39.32 | 24.11 | 15.80 | 10.90 | 7.75 | 5.74 | - | - | - |
| 62 | 364.60 | 163.00 | 82.42 | 46.02 | 27.80 | 17.90 | 12.20 | 8.63 | 6.35 | 5.20 | 4.90 | 4.00 |
| 63 | 461.60 | 200.40 | 99.08 | 54.27 | 32.26 | 20.50 | 13.80 | 9.68 | 7.05 | - | - | - |
| 64 | 591.50 | 249.00 | 120.10 | 64.48 | 37.69 | 23.70 | 15.70 | 10.90 | 7.87 | 6.40 | 5.20 | 4.90 |
| 65 | 767.70 | 313.10 | 147.20 | 77.29 | 44.36 | 27.50 | 17.90 | 12.40 | 8.81 | - | - | - |
| 66 | 1013.00 | 389.50 | 182.00 | 93.45 | 52.61 | 32.10 | 20.60 | 14.10 | 9.93 | 7.70 | 6.40 | 5.80 |
| 67 | 1355.00 | 513.70 | 227.80 | 114.10 | 62.94 | 37.70 | 23.90 | 16.10 | 11.30 | - | - | - |
| 68 | 1846.00 | 672.10 | 288.50 | 140.70 | 75.97 | 44.70 | 27.90 | 18.40 | 12.80 | 8.80 | 7.50 | 6.60 |
| 69 | 2561.00 | 892.50 | 370.10 | 175.60 | 92.58 | 53.00 | 32.90 | 21.40 | 14.70 | - | - | - |
| 70 | 3628.00 | 1206.00 | 481.60 | 221.60 | 114.00 | 64.40 | 39.00 | 25.00 | 16.80 | 9.80 | 8.40 | 7.50 |
| 71 | 5253.00 | 1658.00 | 636.30 | 283.40 | 142.00 | 78.40 | 46.60 | 29.40 | 19.50 | - | - | - |
| 72 | 7792.00 | 2329.00 | 854.90 | 367.60 | 178.90 | 96.50 | 56.10 | 34.90 | 22.80 | - | - | - |
| 73 | 11876.00 | 3340.00 | 1170.00 | 484.30 | 228.50 | 121.00 | 68.40 | 41.70 | 26.90 | - | - | - |
| 74 | 18639.00 | 4906.00 | 1631.00 | 648.50 | 296.00 | 152.00 | 84.10 | 50.30 | 32.00 | - | - | - |
| 75 | 30207.00 | 7401.00 | 2328.00 | 884.80 | 389.50 | 193.00 | 105.00 | 61.40 | 38.30 | - | - | - |
| 76 | - | - | - | 1214.00 | 513.00 | 246.30 | 133.00 | 76.80 | 46.60 | - | - | - |
| 77 | - | - | - | - | 701.00 | 323.40 | 170.70 | 95.10 | 57.20 | - | - | - |
| 78 | - | - | - | - | 980.00 | 433.00 | 221.60 | 121.50 | 71.10 | - | - | - |
| 79 | - | - | - | - | 1430.00 | 598.00 | 292.70 | 156.20 | 89.60 | - | - | - |
| 80 | - | - | - | - | 2160.00 | 835.00 | 393.90 | 204.40 | 115.20 | 83.00 | - | - |

## 3. RESULTS AND DISCUSSIONS

### 3.1 Density

Using Microsoft Excel™ 2007 spreadsheets and CurveExpert® software, a quadratic correlation between sucrose concentration and density, at constant temperature *T*, [K] has been established:

$$\rho = A + BX + CX^2 \qquad (1)$$

The *A*, *B* and *C* values are presented in Table 3. The regression coefficients $R^2$ are greater than 0.99, thus indicating a good correlation of variables.

In order to correlate *A*, *B* and C coefficients with sucrose mass concentration, more models were used in CurveExpert® software ($1^{st}$, $2^{nd}$ and $3^{rd}$ degree polynomial equations, "vapor pressure" model, "heat capacity" model etc.).

The best fit model is the linear equation with good regression coefficients (Table 3).

$$Coefficient = a + bT \qquad (2)$$

Combining the equations 1 and 2 and replacing the coefficients with numeric values, the final form of proposed equation model (Equation 3).

**Table 3. Coefficients for equations 1 and 2**

| Equation 1 | | | | |
|---|---|---|---|---|
| Temperature, T [K] | *A* | *B* | *C* | $R^2$ |
| 283 | 1000.8 | 3.7625 | 0.0181 | 1.000 |
| 293 | 1000.7 | 3.5859 | 0.0194 | 0.999 |
| 303 | 997.05 | 3.6369 | 0.0185 | 1.000 |
| 313 | 993.70 | 3.5994 | 0.0186 | 1.000 |
| 323 | 989.66 | 3.5594 | 0.0188 | 1.000 |
| 333 | 984.72 | 3.5528 | 0.0187 | 1.000 |
| 343 | 980.73 | 3.4174 | 0.0201 | 0.999 |
| 353 | 975.40 | 3.4591 | 0.0194 | 0.999 |
| Equation 2 | | | | |
| Coefficients | *a* | *b* | | $R^2$ |
| *A* | 0.0138771429 | 1.59524E-05 | | 0.373 |
| *B* | 4.8052878571 | -0.00387928 | | 0.803 |
| *C* | 1110.98 | -0.37938095 | | 0.974 |

$$\rho = (0.00015924X + 0.0138771429)T^2 + \\ + (-0.0038792857X + 4.8052878571)T + \\ + (-0.3793809524X + 1110.98) \qquad (3)$$

By plotting tabular data for aqueous sucrose solution in TableCurve 3D® v.4 software (Figure 1 and 2) two equations for the response function were generated, chosen due to the accuracy, respectively, the simplicity of formulation.

The equation (4) is a linear equation, Rank 69, Eqn. 151412160 in TableCurve 3D® v.4 library with a better precision $R^2$ = 0.9998723, $R^2$adj = 0.99986639, FitSdErr = 1.5048729, Fstat. = 203615.33 and equation (5) is a simple equation, Rank 11, Eqn. 302461503 with a simple formulation and $R^2$ = 0.999465, $R^2$adj = 0.9945315, FitSdErr = 3.0447197, Fstat. = 124302.17.

$$\ln\rho = a + bX + cX^{0.5}\ln X + \frac{dX}{\ln X} + \\ + eX^{0.5} + fT^3 \qquad (4)$$

$$\rho^{-1} = a + bX + cT^3 \qquad (5)$$

The coefficients for equations (4) and (5) are presented in Table 4.

**Table 4. Coefficients for equations 4 and 5**

| Equation 4 | | | |
|---|---|---|---|
| Coeff. | *Value* | Coeff. | *Value* |
| *a* | 6.7661174 | *d* | 0.05326616 |
| *b* | 0.003465953 | *e* | 0.055838312 |
| *c* | -0.031302847 | *f* | -1.4054466E-09 |
| Equation 5 | | | |
| Coeff. | *Value* | Coeff. | *Value* |
| *a* | 0.00096991369 | *c* | 1.1322358 E-12 |
| *b* | -3.655077E-06 | | |

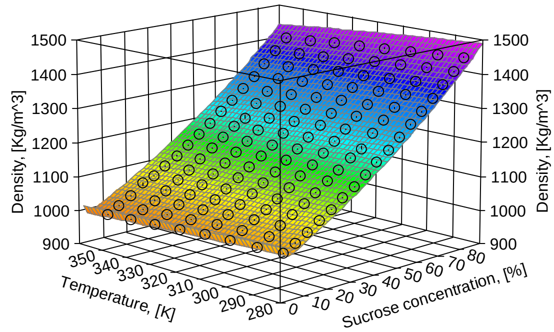


**Figure 1. Aqueous sucrose solutions density values plotted in TableCurve 3D and fitted linear equation (4) with residuals**

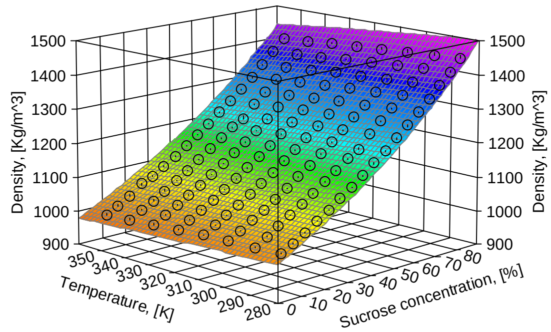


**Figure 2. Aqueous sucrose solutions density values plotted in TableCurve 3D and fitted simple equation (5) with residuals**

To quantify the deviation of calculated densities from tabular data, the relative error equation was use and its values are presented in Table 5:

$$\varepsilon = \left| \frac{\rho_{tabular} - \rho_{calculated}}{\rho_{tabular}} \right| \cdot 100 \quad [\%] \qquad (6)$$

**Table 5. The induced relative errors for the proposed models for density of aqueous sucrose solutions**

| Sucrose concentration, X w [%] | Relative errors, $\varepsilon$ [%] | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Temperature, $T$ [K] | | | | | | | | | | | | | | | |
| | 283 | | 293 | | 303 | | 313 | | 323 | | 333 | | 343 | | 353 | |
| | Equation 3 | Equation 4 | Equation 3 | Equation 4 | Equation 3 | Equation 4 | Equation 3 | Equation 4 | Equation 3 | Equation 4 | Equation 3 | Equation 4 | Equation 3 | Equation 4 | Equation 3 | Equation 4 |
| 5 | 0.299 | 0.215 | 0.082 | 0.039 | 0.038 | 0.064 | 0.077 | 0.109 | 0.046 | 0.110 | 0.049 | 0.072 | 0.202 | 0.003 | 0.411 | 0.095 |
| 10 | 0.229 | 0.178 | 0.025 | 0.026 | 0.090 | 0.062 | 0.131 | 0.099 | 0.108 | 0.097 | 0.023 | 0.059 | 0.117 | 0.008 | 0.311 | 0.100 |
| 15 | 0.178 | 0.183 | 0.012 | 0.053 | 0.122 | 0.021 | 0.163 | 0.050 | 0.144 | 0.043 | 0.069 | 0.007 | 0.058 | 0.057 | 0.235 | 0.142 |
| 20 | 0.142 | 0.151 | 0.033 | 0.043 | 0.136 | 0.018 | 0.177 | 0.038 | 0.162 | 0.029 | 0.095 | 0.008 | 0.022 | 0.068 | 0.758 | 0.796 |
| 25 | 0.120 | 0.108 | 0.041 | 0.018 | 0.137 | 0.028 | 0.177 | 0.042 | 0.165 | 0.030 | 0.104 | 0.006 | 0.003 | 0.063 | 0.154 | 0.136 |
| 30 | 0.106 | 0.062 | 0.040 | 0.007 | 0.128 | 0.041 | 0.165 | 0.049 | 0.155 | 0.033 | 0.101 | 0.001 | 0.003 | 0.053 | 0.135 | 0.119 |
| 35 | 0.099 | 0.022 | 0.033 | 0.030 | 0.112 | 0.052 | 0.146 | 0.053 | 0.138 | 0.036 | 0.089 | 0.003 | 0.001 | 0.044 | 0.127 | 0.101 |
| 40 | 0.095 | 0.013 | 0.022 | 0.048 | 0.093 | 0.059 | 0.124 | 0.055 | 0.116 | 0.036 | 0.244 | 0.178 | 0.007 | 0.019 | 0.126 | 0.084 |
| 45 | 0.092 | 0.041 | 0.039 | 0.011 | 0.075 | 0.064 | 0.101 | 0.054 | 0.094 | 0.034 | 0.052 | 0.006 | 0.022 | 0.029 | 0.126 | 0.067 |
| 50 | 0.089 | 0.062 | 0.737 | 0.669 | 0.056 | 0.063 | 0.079 | 0.050 | 0.072 | 0.030 | 0.034 | 0.005 | 0.033 | 0.022 | 0.127 | 0.050 |
| 55 | 0.083 | 0.077 | 0.006 | 0.072 | 0.042 | 0.060 | 0.061 | 0.044 | 0.054 | 0.025 | 0.020 | 0.005 | 0.778 | 0.752 | 0.124 | 0.031 |
| 60 | 0.075 | 0.085 | 0.009 | 0.070 | 0.032 | 0.053 | 0.049 | 0.036 | 0.041 | 0.019 | 0.011 | 0.005 | 0.043 | 0.007 | 0.116 | 0.011 |
| 65 | 0.064 | 0.084 | 0.008 | 0.062 | 0.027 | 0.043 | 0.042 | 0.026 | 0.036 | 0.014 | 0.009 | 0.005 | 0.039 | 0.002 | 0.101 | 0.012 |
| 70 | 0.052 | 0.073 | 0.003 | 0.048 | 0.027 | 0.026 | 0.042 | 0.013 | 0.037 | 0.005 | 0.014 | 0.004 | 0.026 | 0.013 | 0.079 | 0.036 |
| 75 | 0.040 | 0.050 | 0.003 | 0.022 | 0.034 | 0.006 | 0.099 | 0.150 | 0.045 | 0.005 | 0.027 | 0.003 | 0.006 | 0.023 | 0.049 | 0.060 |
| 80 | 0.029 | 0.012 | 0.011 | 0.013 | 0.038 | 0.029 | 0.058 | 0.028 | 0.056 | 0.024 | 0.048 | 0.001 | 0.025 | 0.034 | 0.013 | 0.080 |
| 85 | 0.023 | 0.044 | 0.015 | 0.067 | 0.052 | 0.067 | 0.069 | 0.064 | 0.079 | 0.042 | 0.068 | 0.016 | 0.057 | 0.036 | 0.038 | 0.108 |

The averages of the induced relative errors for the proposed models are 0.0007% for equation (3), 0.0003% for (4) and 0.0008% for (5).

### 3.2 Dynamic and kinematic viscosity

For calculating the dynamic viscosity of aqueous sucrose solutions the Arrhenius equation form was used because it creates a good correlation between tabular data and calculated values.

$$\mu = \mu_0 \cdot e^{-\frac{E_a}{R \cdot T}} \qquad (7)$$

where: $\mu$ – sugar solutions viscosity [mPa.s]; $\mu_0$ – water viscosity [mPa.s]; $Ea$ – activation energy [kcal/mol]; $R$ – universal gas constant [$1.987\times10^3$ kcal/mol]; $T$ – absolute temperature [K]

Taking logs of equation (9), it gets:

$$\log\mu = \log\mu_0 - \frac{E_a}{2.303 \cdot R} \cdot \frac{1}{T} \qquad (8)$$

and:

$$\frac{E_a}{R} = 2.303(\log\mu_0 - \log\mu)T \qquad (9)$$

By plotting in TableCurve 3D® v.4 software the results obtained from equation (9) (Figure 3) which make correlations between tabular data from Table 2 and water viscosity an equation for the response surface was generated (10).

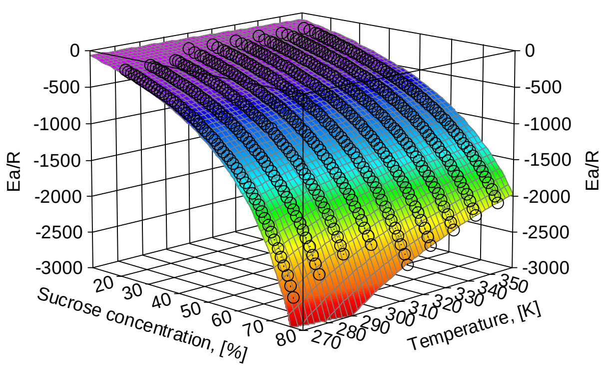


**Figure 3. *Ea/R* values plotted in TableCurve 3D and fitted linear equation (9) with residuals**

$$\frac{E_a}{R} = \frac{a + bX + cX^2 + dX^3 + e\ln T + f(\ln T)^2}{1 + gX + hX^2 + i\ln T} \qquad (10)$$

The equation (10) is a linear equation, Rank 34, Eqn. 1124 in TableCurve 3D® v.4 library with a better precision $R^2$ = 0.99998026, $R^2$adj = 0.99997993, FitSdErr = 2.6506519, Fstat. = 3375314.1.

**Table 6. Coefficients for equation (10)**

| Coeff. | *Value* | Coeff. | *Value* |
|---|---|---|---|
| *a* | 999.26172 | *f* | 35.811643 |
| *b* | 1.4438886 | *g* | -0.00083179422 |
| *c* | 0.015074345 | *h* | 2.1503594E-05 |
| *d* | 4.6988139E-05 | *i* | -0.20243452 |
| *e* | -380.31333 | | |

To quantify the deviation from experimental data, between measured and calculated dynamic viscosity, the relative error was used:

$$\varepsilon = \left| \frac{\mu_{measured} - \mu_{calculated}}{\mu_{measured}} \right| \cdot 100 \quad [\%] \quad (11)$$

By analyzing the obtained values (Table 7) it can be observed that the induced relative error for the proposed equation model is only 0.011% for an interval of temperature between 0 and 80 °C. If the average is calculated for range of 0 to 100 °C the value increases to 0.11%.

**Table 7. The induced relative errors for the proposed models for dynamic viscosity of aqueous sucrose solutions**

| Sucrose concentration, *X* w [%] | Relative errors, *ε* [%] | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Temperature, *t* [°C] | | | | | | | | | | | |
| | 273 | 283 | 293 | 303 | 313 | 323 | 333 | 343 | 353 | 363 | 368 | 373 |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| 16 | - | - | - | 1.408 | 0.219 | 0.280 | 1.605 | 1.307 | 2.429 | 6.054 | - | 8.756 |
| 18 | - | - | - | 0.621 | 1.267 | 0.808 | 1.448 | 0.051 | 2.567 | 4.629 | - | 3.942 |
| 20 | 3.115 | 2.682 | 1.743 | 0.940 | 0.025 | 0.001 | 0.224 | 1.901 | 1.506 | 0.987 | - | 0.815 |
| 21 | 2.718 | 2.358 | 1.468 | 0.741 | 0.057 | 0.671 | 0.068 | 0.842 | 1.254 | - | - | - |
| 22 | 2.316 | 2.060 | 1.242 | 0.597 | 0.058 | 0.542 | 0.115 | 1.414 | 1.179 | - | - | - |
| 23 | 1.943 | 1.755 | 1.033 | 0.363 | 0.309 | 0.277 | 0.360 | 0.797 | 1.276 | - | - | - |
| 24 | 1.555 | 1.473 | 0.811 | 0.252 | 0.367 | 0.084 | 0.800 | 0.416 | 1.543 | - | - | - |
| 25 | 1.235 | 1.236 | 0.630 | 0.169 | 0.416 | 0.664 | 0.388 | 0.262 | 0.520 | - | - | - |
| 26 | 0.889 | 0.972 | 0.493 | 0.084 | 0.480 | 0.631 | 0.236 | 0.325 | 1.153 | - | - | - |
| 27 | 0.565 | 0.751 | 0.323 | 0.017 | 0.455 | 0.078 | 0.334 | 0.599 | 0.592 | - | - | - |
| 28 | 0.224 | 0.525 | 0.192 | 0.066 | 0.441 | 0.149 | 0.241 | 0.017 | 0.279 | - | - | - |
| 29 | 0.075 | 0.318 | 0.090 | 0.151 | 0.465 | 0.069 | 0.365 | 0.348 | 0.201 | - | - | - |
| 30 | 0.338 | 0.145 | 0.061 | 0.146 | 0.388 | 0.375 | 0.483 | 0.409 | 0.828 | 2.091 | - | 2.094 |
| 31 | 0.692 | 0.052 | 0.150 | 0.184 | 0.415 | 0.338 | 0.177 | 0.211 | 0.418 | - | - | - |
| 32 | 1.008 | 0.249 | 0.262 | 0.261 | 0.416 | 0.026 | 0.363 | 0.678 | 0.882 | - | - | - |
| 33 | 1.279 | 0.415 | 0.341 | 0.306 | 0.339 | 0.114 | 0.223 | 0.827 | 1.055 | - | - | - |
| 34 | 1.482 | 0.553 | 0.424 | 0.339 | 0.370 | 0.005 | 0.465 | 0.673 | 0.952 | - | - | - |
| 35 | 1.623 | 0.644 | 0.476 | 0.380 | 0.367 | 0.270 | 0.344 | 1.022 | 1.528 | - | - | - |
| 36 | 1.913 | 0.790 | 0.524 | 0.332 | 0.342 | 0.096 | 0.505 | 1.021 | 1.765 | - | - | - |
| 37 | 1.987 | 0.857 | 0.560 | 0.354 | 0.310 | 0.002 | 0.276 | 0.685 | 1.682 | - | - | - |
| 38 | 2.117 | 0.901 | 0.566 | 0.365 | 0.292 | 0.073 | 0.241 | 0.717 | 1.290 | - | - | - |
| 39 | 2.263 | 0.956 | 0.596 | 0.362 | 0.341 | 0.248 | 0.333 | 1.031 | 1.384 | - | - | - |
| 40 | 2.135 | 0.920 | 0.603 | 0.397 | 0.353 | 0.043 | 0.487 | 0.926 | 1.122 | 0.475 | - | 2.623 |
| 41 | 2.019 | 0.823 | 0.553 | 0.393 | 0.429 | 0.099 | 0.166 | 0.421 | 1.229 | - | - | - |
| 42 | 1.792 | 0.662 | 0.508 | 0.384 | 0.438 | 0.042 | 0.301 | 0.095 | 0.941 | - | - | - |
| 43 | 1.566 | 0.599 | 0.441 | 0.354 | 0.444 | 0.075 | 0.336 | 0.413 | 0.918 | - | - | - |
| 44 | 1.402 | 0.474 | 0.376 | 0.340 | 0.460 | 0.090 | 0.230 | 0.217 | 1.078 | - | - | - |
| 45 | 1.170 | 0.336 | 0.309 | 0.323 | 0.465 | 0.124 | 0.057 | 0.008 | 0.191 | - | - | - |
| 46 | 0.944 | 0.249 | 0.184 | 0.294 | 0.480 | 0.099 | 0.146 | 0.180 | 0.566 | - | - | - |
| 47 | 0.821 | 0.121 | 0.176 | 0.253 | 0.477 | 0.008 | 0.016 | 0.184 | 0.395 | - | - | - |
| 48 | 0.675 | 0.034 | 0.133 | 0.236 | 0.472 | 0.005 | 0.148 | 0.020 | 0.196 | - | - | - |
| 49 | 0.519 | 0.096 | 0.038 | 0.205 | 0.467 | 0.065 | 0.119 | 0.099 | 0.371 | - | - | - |
| 50 | 0.340 | 0.267 | 0.016 | 0.143 | 0.455 | 0.148 | 0.129 | 0.155 | 0.357 | 1.629 | - | 0.193 |
| 51 | 0.194 | 0.314 | 0.100 | 0.125 | 0.416 | 0.149 | 0.066 | 0.037 | 0.113 | - | - | - |
| 52 | 0.080 | 0.391 | 5.281 | 0.107 | 0.402 | 0.121 | 0.060 | 0.037 | 0.349 | - | - | - |
| 53 | 0.005 | 0.460 | 0.207 | 0.018 | 0.371 | 0.065 | 0.182 | 0.053 | 0.192 | - | - | - |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 54 | 0.104 | 0.584 | 0.302 | 0.008 | 0.279 | 0.011 | 0.109 | 0.032 | 0.267 | - | - | - |
| 55 | 0.199 | 0.618 | 0.307 | 0.067 | 0.327 | 0.012 | 0.005 | 0.086 | 0.126 | - | - | - |
| 56 | 0.639 | 0.682 | 0.381 | 0.109 | 0.248 | 0.020 | 0.047 | 0.037 | 0.253 | - | - | - |
| 57 | 0.347 | 0.746 | 0.415 | 0.174 | 0.203 | 0.061 | 0.028 | 0.248 | 0.212 | - | - | - |
| 58 | 0.430 | 0.797 | 0.482 | 0.209 | 0.162 | 0.691 | 0.135 | 0.109 | 0.137 | - | - | - |
| 59 | 0.364 | 0.860 | 0.525 | 0.237 | 0.169 | 0.314 | 0.065 | 0.205 | 0.104 | - | - | - |
| 60 | 0.474 | 0.970 | 0.604 | 0.294 | 0.130 | 0.524 | 0.012 | 0.088 | 0.252 | 2.277 | - | 0.497 |
| 61 | 0.598 | 1.064 | 0.697 | 0.361 | 0.117 | 0.194 | 0.674 | 0.160 | 0.143 | - | - | - |
| 62 | 0.680 | 1.199 | 0.781 | 0.409 | 0.084 | 0.284 | 0.057 | 0.368 | 0.200 | 7.246 | - | 5.929 |
| 63 | 0.821 | 1.255 | 0.887 | 0.479 | 0.054 | 0.244 | 0.011 | 0.301 | 0.309 | - | - | - |
| 64 | 0.884 | 1.295 | 0.911 | 0.514 | 0.030 | 0.136 | 0.038 | 0.325 | 0.304 | 7.919 | - | 7.428 |
| 65 | 0.833 | 1.370 | 1.062 | 0.589 | 0.001 | 0.239 | 0.421 | 0.165 | 0.479 | - | - | - |
| 66 | 0.922 | 0.872 | 1.034 | 0.627 | 0.010 | 0.251 | 0.476 | 0.066 | 0.457 | 4.916 | - | 4.281 |
| 67 | 0.728 | 1.360 | 1.090 | 0.671 | 0.051 | 0.143 | 0.370 | 0.194 | 0.005 | - | - | - |
| 68 | 0.571 | 1.325 | 1.092 | 0.660 | 0.077 | 0.198 | 0.340 | 0.985 | 0.547 | 5.319 | - | 4.721 |
| 69 | 0.264 | 1.174 | 1.045 | 0.720 | 0.089 | 0.592 | 0.043 | 0.677 | 0.309 | - | - | - |
| 70 | 0.107 | 1.016 | 0.975 | 0.667 | 0.094 | 0.163 | 0.017 | 0.630 | 1.194 | 22.283 | - | 16.952 |
| 71 | 0.669 | 0.673 | 0.849 | 0.620 | 0.119 | 0.129 | 0.051 | 0.674 | 1.192 | - | - | - |
| 72 | 1.435 | 0.384 | 0.678 | 0.548 | 0.057 | 0.141 | 0.254 | 0.589 | 1.178 | - | - | - |
| 73 | 2.394 | 0.143 | 0.471 | 0.467 | 0.038 | 0.863 | 0.177 | 0.723 | 1.030 | - | - | - |
| 74 | 3.660 | 0.823 | 0.039 | 0.299 | 0.011 | 0.541 | 0.330 | 0.844 | 0.886 | - | - | - |
| 75 | 5.342 | 1.661 | 0.303 | 0.133 | 0.064 | 0.049 | 0.106 | 0.782 | 1.023 | - | - | - |
| 76 | - | - | - | 1.539 | 1.759 | 1.606 | 0.307 | 0.628 | 0.471 | - | - | - |
| 77 | - | - | - | - | 1.634 | 1.895 | 0.674 | 0.149 | 0.076 | - | - | - |
| 78 | - | - | - | - | 1.289 | 1.968 | 0.701 | 0.912 | 0.455 | - | - | - |
| 79 | - | - | - | - | 1.061 | 0.826 | 0.838 | 1.204 | 1.140 | - | - | - |
| 80 | - | - | - | - | 4.264 | 0.694 | 1.071 | 1.743 | 2.491 | 18.813 | - | - |

Kinematic viscosity (ν) [$m^2/s$] of aqueous sucrose solutions can be calculated using models developed for the calculation of density (Equations 3, 4 and 5) and dynamic viscosity (Equation 10).

$$\nu = \frac{\mu}{\rho} \qquad (12)$$

## 4. CONCLUSIONS

Three equations were generated for density and one for viscosity, taking in consideration the level of precision and simplicity of formulation.

From the existing data for density of aqueous sucrose solutions one equation with an average of relative errors of 0.00011% (Equation 3) and two equations with an regression coefficients greater than 0.999 (Equations 4 and 5) were generated, for ranges of 5 – 85% in sucrose concentration and temperature of 10 – 80 °C.

For dynamic viscosity an equations was formulated with average relative errors of 0.11% (Equation 10) for intervals of temperature of 0 to 100 °C and sucrose concentration of 16 to 80%.

The obtained equations can be loaded in computer software available both for industrial and academic users and so facilitating the sizing and optimization calculations of various technological equipment and processes.

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